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LATTICE QCD AT NON-ZERO BARYON NUMBER

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We discuss the quenched limit of lattice QCD at non-zero baryon number density. We find evidence for a mixed phase that becomes broader with increasing baryon number. Although the action is explicitly $Z(3)$ symmetric the Polyakov loop expectation value becomes non-zero already in the low temperature phase. It indicates that the heavy quark potential stays finite at large distances, i.e. the string between static quarks breaks at non-zero baryon number density already in the hadronic phase. This behaviour is validated by calculating the heavy quark potential using Polyakov loop correlations.

1 Introduction

A quantitative analysis of QCD at non-zero density is important for our understanding of the behaviour of dense matter as it is created in heavy ion collisions and exists in the cosmological context. While the QCD phase diagram is well known for vanishing baryon density from lattice QCD, for the region of non-zero density only qualitative features can be understood in terms of models (bags, percolation, strings, ...) and approximations (resonance gas, perturbation theory, instanton liquid, ...). Non-zero baryon number is usually introduced by a non-zero chemical potential μ[1,2]. This leads to a break down of the probabilistic interpretation of the path integral representation of the QCD partition function as the fermion determinant gets complex[3].
The static limit of QCD at non-zero chemical potential μ has been formulated by Bender et al.[4] and Blum et al.[5]. It seems that in this case the first order deconfinement transition of the $SU(3)$ gauge theory turns into a crossover for arbitrarily small, non-zero values of the chemical potential[5]. Rather than introducing a non-vanishing chemical potential, i.e. formulate QCD at non-vanishing baryon number density in the grand canonical ensemble, one may go over to a canonical formulation of the thermodynamics and fix directly the baryon number[6]. This is achieved by introducing an imaginary chemical potential[6,7] in the grand canonical partition function and performing a Fourier integration to project onto the canonical partition function for a given sector of fixed baryon number[6]

$$Z(B,T,V) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi e^{-iB\phi} Z(i\phi,T,V). \tag{1}$$

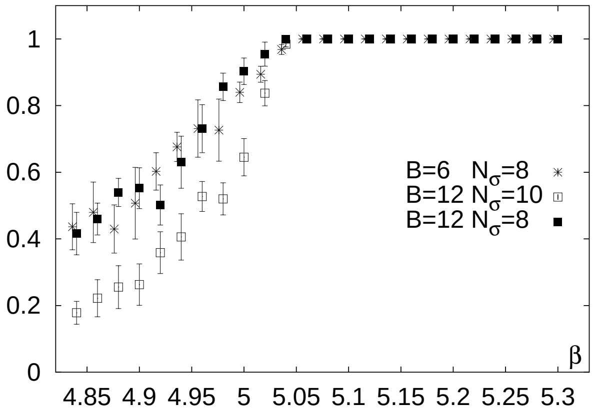

Figure 1. $\langle \mathrm{sgn}(\mathrm{Re}\hat{f}_B)\rangle_{||}$ for $B = 6$ and 12 and lattices of size $8^3 \times 2$ and $10^3 \times 2$.

This leads to a well defined quenched theory[8] at fixed baryon number, which describes the thermodynamics of gluons in the background of a non-zero number of static quark sources, suitably arranged to obey Fermi statistics.

2 Simulation of quenched QCD with non-zero baryon number

For any fixed value of the baryon number we can write the quenched partition function as

$$Z(B,T,V) = \int \prod_{x,\nu} \mathrm{d}U_{x,\nu} \hat{f}_B e^{-S_G} \tag{2}$$

where the constraint on the baryon number is encoded in the function $\hat{f}_B$ which is a function of Polyakov loops and B counts the number of quarks, i.e. $B/3$ is the baryon number. For $B = 3$ $\hat{f}_B$ is, for instance, given by

$$\begin{aligned} f_{B=3} = (2\kappa)^{3N_\tau}(&V^3 \frac{4}{3}[L_{1,0}]^3 \\ &+V^2(8[L_{1,0}][L_{2,0}] - 4[L_{1,0}][L_{0,1}]) \\ &+V(12 + \frac{2}{3}[L_{3,0}] - 2[L_{1,1}])) \end{aligned} \tag{3}$$

with $[L_{i,j}] = V^{-1}\sum_{\vec{x}}(\mathrm{Tr}L_{\vec{x}})^i(\mathrm{Tr}L^2_{\vec{x}})^j$.

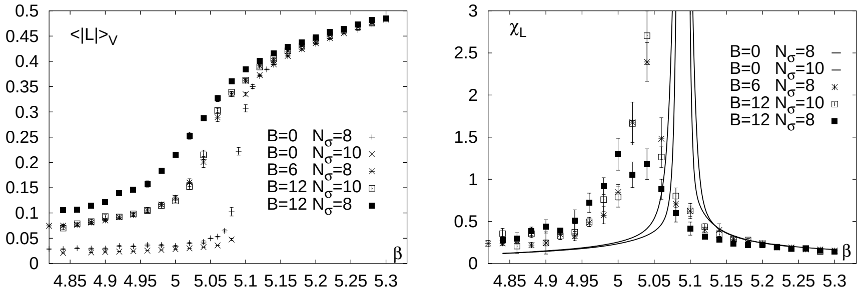

Figure 2. Polyakov loop expectation value (left) and Polyakov loop susceptibility (right) for different values of B and lattices with spatial extend $N_\sigma = 8$ and 10.

For a more detailed description of $\hat{f}_B$ see Engels et al.[8]. S_G is the gluonic action, which is $Z(3)$ symmetric. The partition function $Z(B,T,V)$, is non-zero only if B is a multiple of 3, because $\hat{f}_B$ is invariant under $Z(3)$ transformations only if B is a multiple of 3. In general it changes by a factor $e^{2\pi i B/3}$ under a global $Z(3)$ transformation of timelike link variables.

$\hat{f}_B$ is still a complex function, but upon integration over the gauge fields the imaginary part of the partition function vanishes. The remaining sign problem can be handled by including the sign in the calculation of observables[9]. We have performed simulations for the one flavour case ($n_f = 1$) using the partition function

$$Z_{||}(B,T,V) = \int \prod_{x,\nu} \mathrm{d}U_{x,\nu} |\mathrm{Re}\hat{f}_B| e^{-S_G}. \tag{4}$$

Expectation values of an observable $\mathcal{O}$ will be calculated according to

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} \cdot \mathrm{sgn}(\mathrm{Re}\hat{f}_B)\rangle_{||}}{\langle \mathrm{sgn}(\mathrm{Re}\hat{f}_B)\rangle_{||}}. \tag{5}$$

The simulations are performed on $N_\sigma^3 \times N_\tau$ lattices with $N_\sigma = 8, 10$ and $N_\tau = 2$ using the standard Wilson action with quark number values of $B = 6$ and 12. When increasing the gauge coupling β the temperature is increased while $n_B/T^3 = \frac{1}{3}B(\frac{N_\sigma}{N_\tau})^3$ is kept fixed. Close to T_c a simulation on a $8^3 \times 2$ lattice with $B = 12$ corresponds to $n_B \simeq 0.15/fm^3$, i.e. approximately nuclear matter density. Figure 1 shows the average sign $\langle \mathrm{sgn}(\hat{f}_B)\rangle_{||}$ as a function of the coupling β. For large values of the temperature the sign is almost always positive, but also for the smallest temperature in our analysis the sign can be

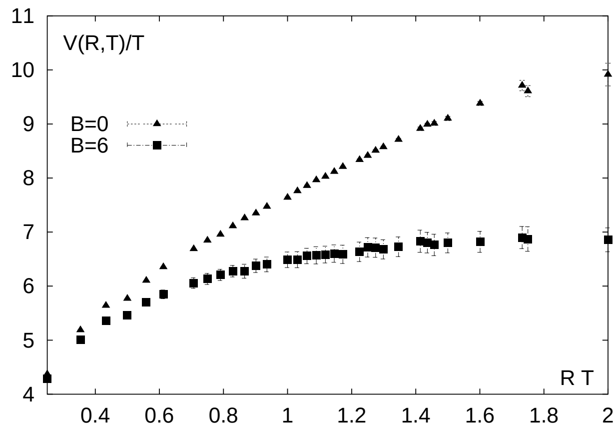

Figure 3. Heavy quark potential for $T \simeq 0.86T_c$ and $B = 0$ and 6.

well determined. It depends on the spatial volume N_σ^3 but varies little with B.

In the quenched limit at vanishing baryon number the Polyakov loop expectation $\langle L \rangle$ value is an order parameter for the deconfinement transition in the infinite volume limit. For a first order phase transition, $\langle L \rangle$ changes discontinuously at T_c. In Figure 2 we see a clear signal for a first order transition for the $B = 0$ case, while for all $B > 0$ the transition is continuous. The transition region is shifted towards smaller values and it broadens with increasing B. This is the expected behaviour for a canonical calculation. By changing the gauge coupling β we vary the lattice cut-off and through this also the baryon number density continuously. At fixed baryon number we therefore follow a simulation path that traverses the mixed phase continuously.
If there is a mixed phase in the sense that there exist singularities in thermodynamic observables when entering and leaving this mixed phase, it could be reflected in a discontinuous change of the slope of the Polyakov loop expectation value. We analyzed this by calculating the conventional Polyakov loop susceptibility χ_L (Figure 2). This response function reflects the existence of a transition region that becomes broader with increasing n_B, but does not show indications for a discontinuity.

The Polyakov loop expectation value becomes non-zero already in the low temperature phase. This indicates that the heavy quark potential stays finite at large distances. We validate this by calculating the potential using

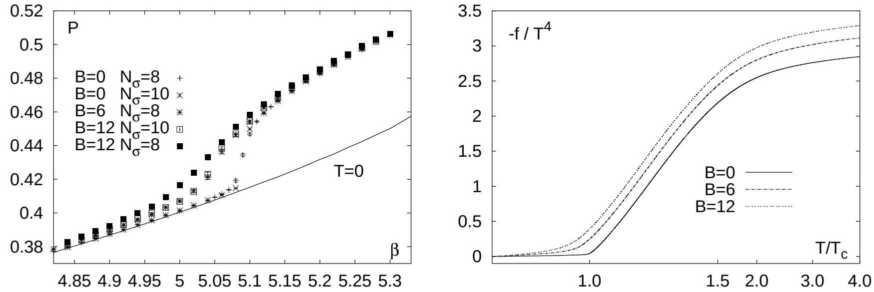

Figure 4. Plaquette expectation value (left) and the negative of the free energy density (right). On the right plot we only show tick marks at critical couplings on lattices with extend $N_\tau = 2, 3, 4, 6$ and 8, which corresponds here to temperatures $T/T_c = 1, 1.5, 2, 3, 4$

Polyakov loop correlations. The potentials for $T \simeq 0.86T_c$ at $B = 0$ and 6 on a $16^3 \times 4$ lattice are plotted in Figure 3. For zero baryon number it shows the usual behaviour for the quenched case. The potential is linearly rising for large distances. For $B = 6$ the potential stays finite at large distances. The static quark anti-quark sources used to probe the heavy quark potential can recombine with the already present static quarks. This leads to string breaking even in the low temperature phase similarly to full QCD[10].

The plaquette expectation value (Figure 4) shows a similar behaviour as the Polyakov loop. With increasing n_B the transition region broadens. Through an integration over differences of plaquette expectation values for finite temperature and zero temperature, the free energy density can be calculated. As the area between these data increases with increasing n_B, the free energy density decreases at fixed temperature with increasing B.

3 Conclusions

We have analyzed the quenched limit of QCD at non-zero baryon number. Although a sign problem remains in this theory, it can be handled quite well numerically. We find indications for a mixed phase, which broadens with increasing baryon number density and is shifted towards smaller temperatures. We also see evidence that the heavy quark potential stays finite for large distances already in the hadronic phase. String breaking starts at short distances.

Acknowledgments

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